Let $f(x) = \cos 2x$. **SCORE: _____ / 9 PTS**

[a] Find the first 3 **non-zero** terms of the Taylor series for f about $x = \frac{\pi}{4}$.

n	$f^{(n)}(x)$	$f^{(n)}(\frac{\pi}{4})$
0	$\cos 2x$	0
1	$-2\sin 2x$	-2
2	$-4\cos 2x$	0
3	$8\sin 2x$	8
4	$16\cos 2x$	0
5	$-32\sin 2x$	-32

1

$$-2(x-\tfrac{\pi}{4}) + \frac{8}{3!}(x-\tfrac{\pi}{4})^3 - \frac{32}{5!}(x-\tfrac{\pi}{4})^5$$

$$= \underbrace{-2(x-\tfrac{\pi}{4})}_{1} + \underbrace{\frac{4}{3}(x-\tfrac{\pi}{4})^3}_{1} - \underbrace{\frac{4}{15}(x-\tfrac{\pi}{4})^5}_{1}$$

[b] Write the Taylor series for f about $x = \frac{\pi}{4}$ in sigma notation.

$$1\ \sum_{n=1}^{\infty} \frac{(-1)^n 2^{2n-1} (x-\frac{\pi}{4})^{2n-1}}{(2n-1)!} \quad 1\ 1\ 1\ 1 \qquad \text{OR} \qquad \sum_{n=0}^{\infty} \frac{(-1)^{n+1} 2^{2n+1} (x-\frac{\pi}{4})^{2n+1}}{(2n+1)!}$$

Use partial fractions decomposition to find a power series for $f(x) = \dfrac{3}{x^2 - x - 2}$. **SCORE: _____ / 6 PTS**

Do **NOT** use the binomial series.

$$\frac{3}{x^2-x-2} = \frac{1}{x-2} - \frac{1}{x+1} \quad 1$$

$$= 1\ \ -\frac{1}{2}\cdot\frac{1}{1-\frac{x}{2}} - \frac{1}{1+x}$$

$$= 1\tfrac{1}{2}\ \ \sum_{n=0}^{\infty} -\frac{1}{2}\left(\frac{x}{2}\right)^n - \sum_{n=0}^{\infty}(-x)^n \quad 1\tfrac{1}{2}$$

$$= \sum_{n=0}^{\infty}\left(-\frac{1}{2^{n+1}} - (-1)^n\right)x^n \quad 1$$

Use the binomial series to write the Maclaurin series for $f(x) = \dfrac{1}{\sqrt[3]{1+x^2}}$ in sigma notation. **SCORE: _____ / 6 PTS**

Simplify all coefficients as shown in lecture.

① BONUS IF YOU REALIZED THIS TERM DID NOT FOLLOW THE PATTERN IN THE Σ

$$f(x) = (1+x^2)^{-\frac{1}{3}} \underset{①}{=} \underbrace{\sum_{n=0}^{\infty} \binom{-\frac{1}{3}}{n} (x^2)^n}_{} = 1 + \underbrace{\sum_{n=1}^{\infty}}_{①} \underbrace{(-1)^n \frac{1 \cdot 4 \cdot 7 \cdots (3n-2)}{3^n \cdot n!}}_{②} \underbrace{x^{2n}}_{①}$$

$$\binom{-\frac{1}{3}}{n} \underset{①}{=} \boxed{\frac{(-\frac{1}{3})(-\frac{4}{3})(-\frac{7}{3}) \cdots (-\frac{1}{3}-n+1)}{n!}} \quad \text{IF } n \geq 1$$

$$= \frac{(-1)^n (1 \cdot 4 \cdot 7 \cdots (3n-2))}{3^n \cdot n!}$$

IF YOU WROTE $\sum_{n=0}^{\infty}$ AND DID NOT GET THE BONUS POINT ABOVE, YOU STILL GET THIS POINT

Consider the series $\displaystyle\sum_{n=1}^{\infty} \frac{3^n (x+1)^n}{\sqrt{n}}$. **SCORE: _____ / 9 PTS**

[a] Find the radius of convergence.

$$\lim_{n\to\infty} \left| \frac{3^{n+1}(x+1)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{3^n (x+1)^n} \right| = \underbrace{3|x+1| \lim_{n\to\infty} \left| \sqrt{\frac{n}{n+1}} \right|}_{①} = \underbrace{3|x+1| < 1}_{①}$$

$$|x+1| < \frac{1}{3}$$

RADIUS $= \underbrace{\frac{1}{3}}_{}$ ①

[b] Find the interval of convergence.

$$|x+1| < \frac{1}{3} \longrightarrow -\frac{1}{3} < x+1 < \frac{1}{3} \longrightarrow -\frac{4}{3} < x < -\frac{2}{3} \quad ①$$

$$x = -\frac{4}{3}: \sum_{n=1}^{\infty} \frac{3^n(-\frac{1}{3})^n}{\sqrt{n}} = \underbrace{\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}}_{} \; ①$$

$\frac{1}{\sqrt{n}} > 0$, $\left\{\frac{1}{\sqrt{n}}\right\}$ IS DECREASING

$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

(½) SERIES CONVERGES BY ALTERNATING SERIES TEST

$$x = -\frac{2}{3}: \sum_{n=1}^{\infty} \frac{3^n(\frac{1}{3})^n}{\sqrt{n}} \underset{(\frac{1}{2})}{=} \underbrace{\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}}_{} \; \underbrace{\text{DIVERGES } (p = \tfrac{1}{2} < 1)}_{} \; (\tfrac{1}{2})$$

$$\text{INTERVAL} = \left[-\frac{4}{3}, -\frac{2}{3}\right) \text{ OR } \underbrace{-\frac{4}{3}}_{(\frac{1}{2})} \underbrace{\leq}_{①} \underbrace{x}_{(\frac{1}{2})} \underbrace{< -\frac{2}{3}}_{(\frac{1}{2})}$$